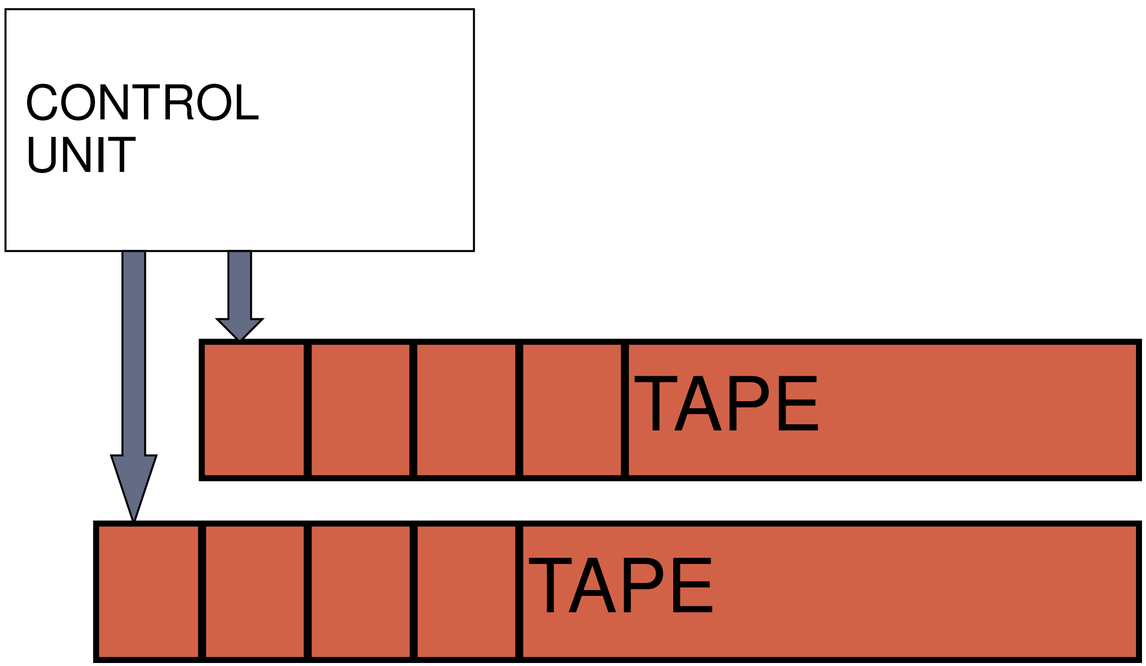
Lecture 26

Multitape TM

Multitape TM

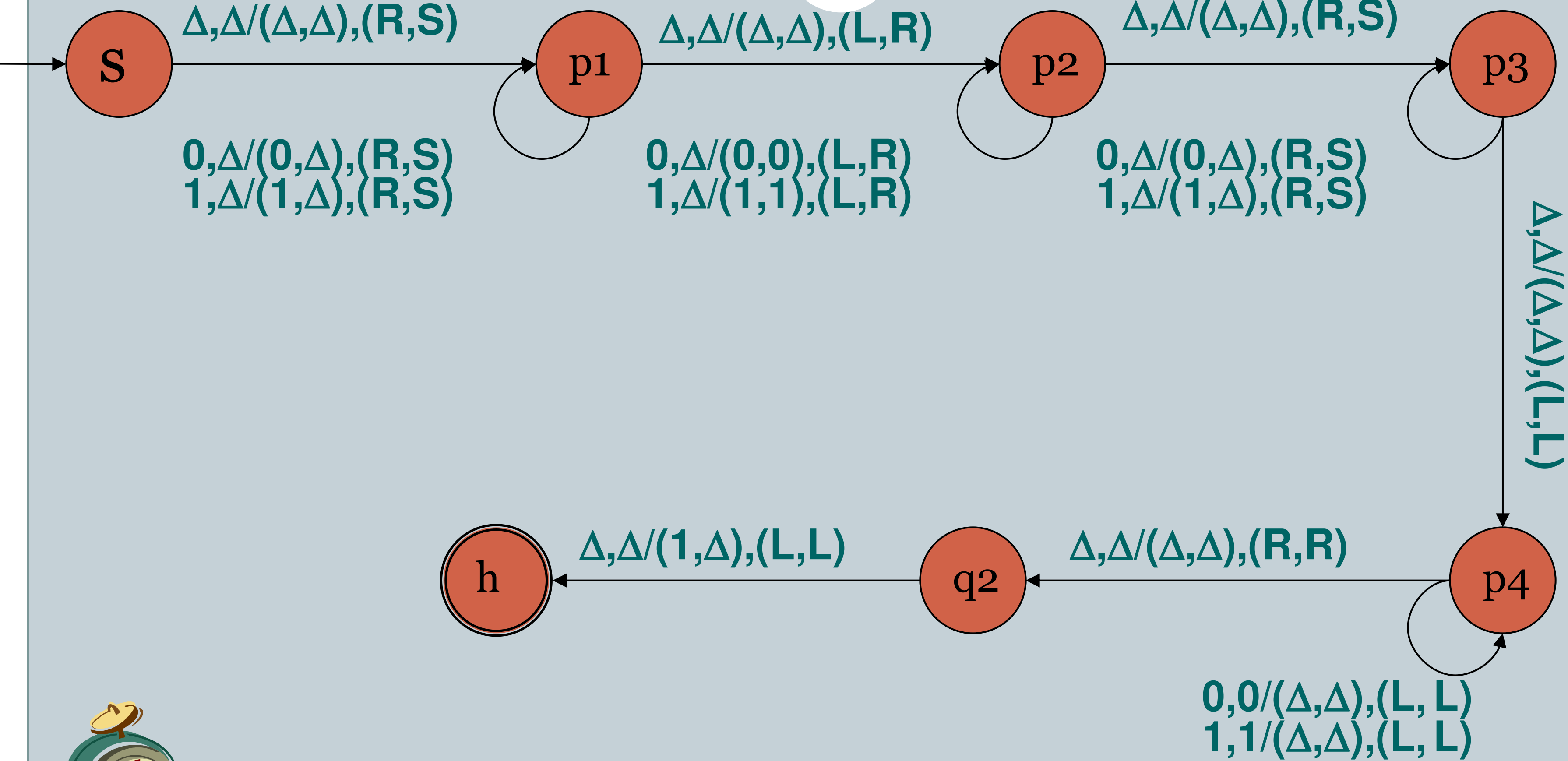
- TM with more than one tape.
- Each tape has its own tape head.
- Each tape is independent.



2-Tape Turing Machine

- a quintuple $(Q, \Sigma, \Gamma, \delta, s)$, where
  - the set of states Q is finite, and does not contain the halt state h,
  - the input alphabet Σ is a finite set of symbols, not including the blank symbol Δ,
  - the tape alphabet Γ is a finite set of symbols containing Σ, but not including the blank symbol Δ,
  - the start state s is in Q, and
  - the transition function δ is a partial function from $Q \times (\Gamma \cup \{\Delta\})^2 \rightarrow Q \cup \{h\} \times (\Gamma \cup \{\Delta\})^2 \times \{L, R, S\}^2$.

Example of 2-Tape Turing Machine



Equivalence of 2-tape TM and single-tape TM

Theorem:

For any 2-tape TM T, there exists a single-tape TM M such that for any string α in Σ^*:

- if T halts on α with β on its tape, then M halts on α with β on its tape, and
- if T does not halt on α, then M does not halt on α.

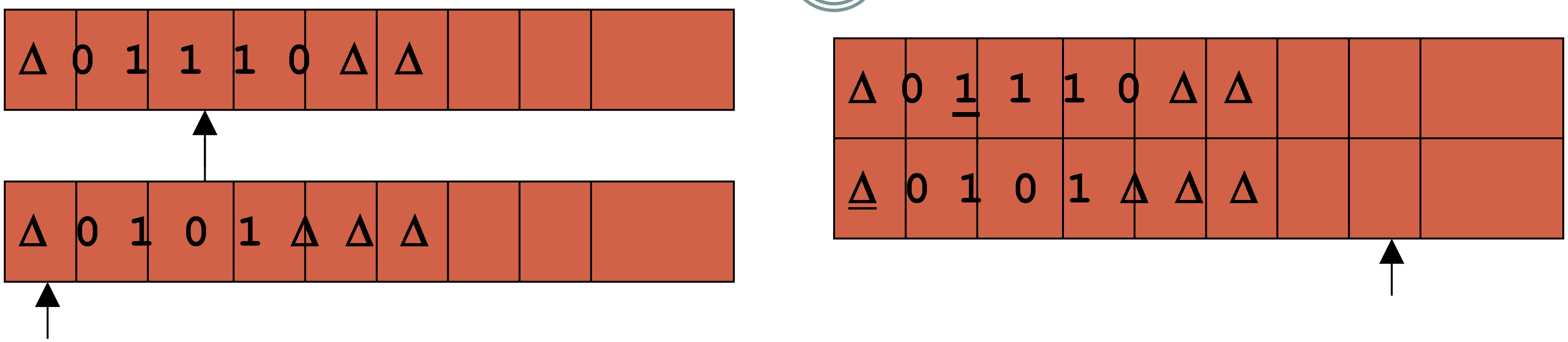
How 1-tape TM simulates 2-tape TM

- Marking the position of each tape head in the content of the tape
- Encode content of 2 tapes on 1 tape
  - When to convert 1-tape symbol into 2-tape symbol

cannot be done all at once because the tape is infinite

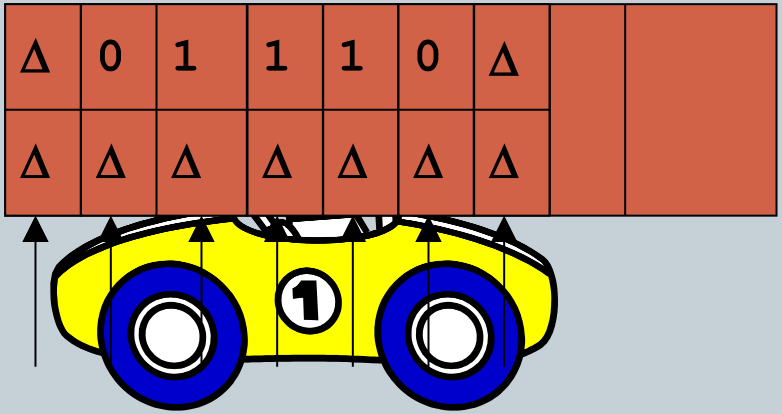
- Construct 1-tape TM simulating a transition in 2-tape TM
- Convert the encoding of 2-tape symbols back to 1-tape symbols

Encoding 2 tapes in 1 tape

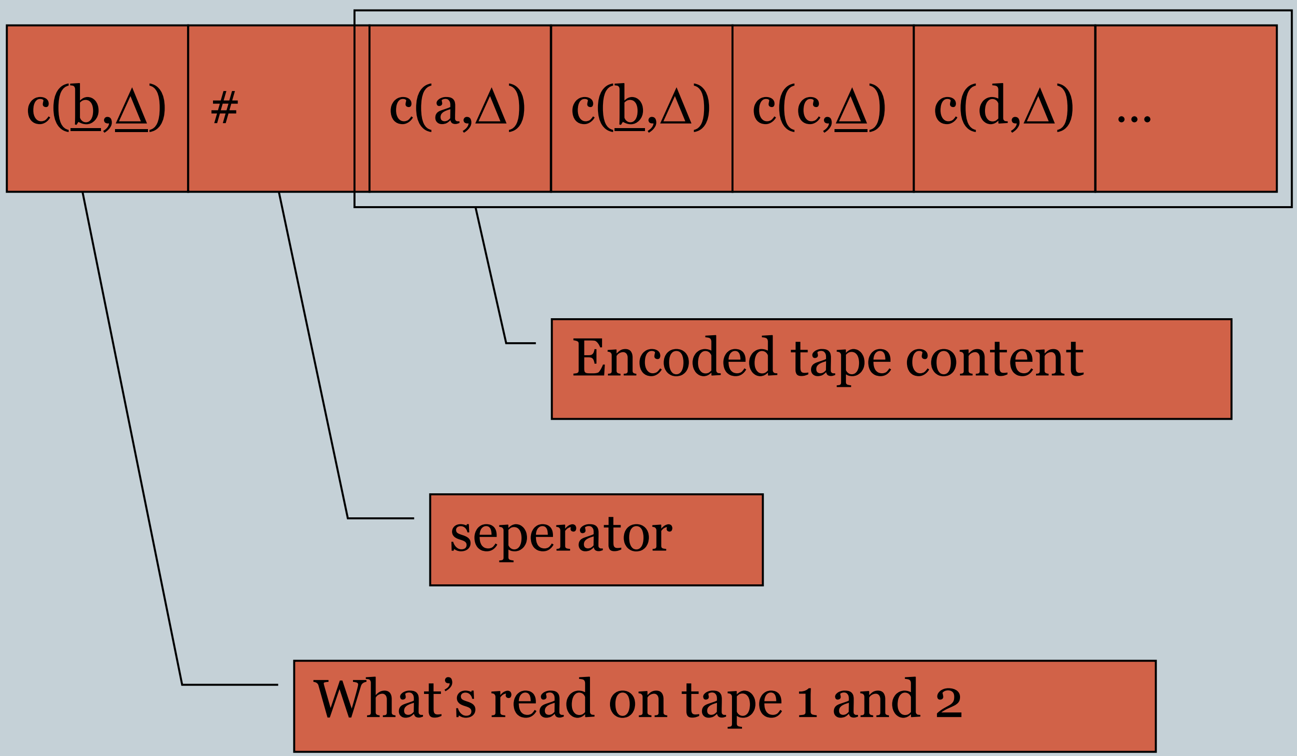


- New alphabet contains:
  - old alphabet
  - encoding of a symbol on tape 1 and a symbol on tape 2
  - encoding of a symbol on tape 1 pointed by its tape head and a symbol on tape 2
  - encoding of a symbol on tape 1 and a symbol on tape 2 pointed by its tape head
  - encoding of a symbol on tape 1 pointed by its tape head and a symbol on tape 2 pointed by its tape head

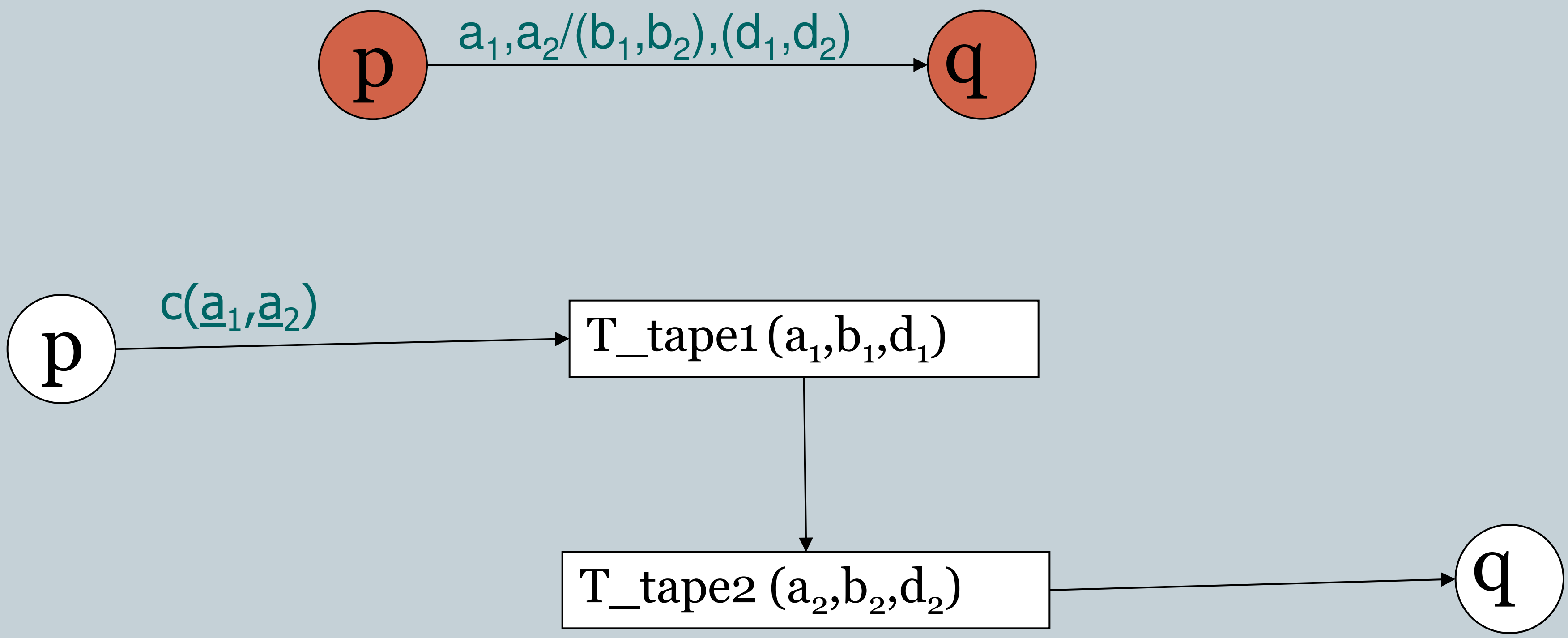
How the tape content is changed



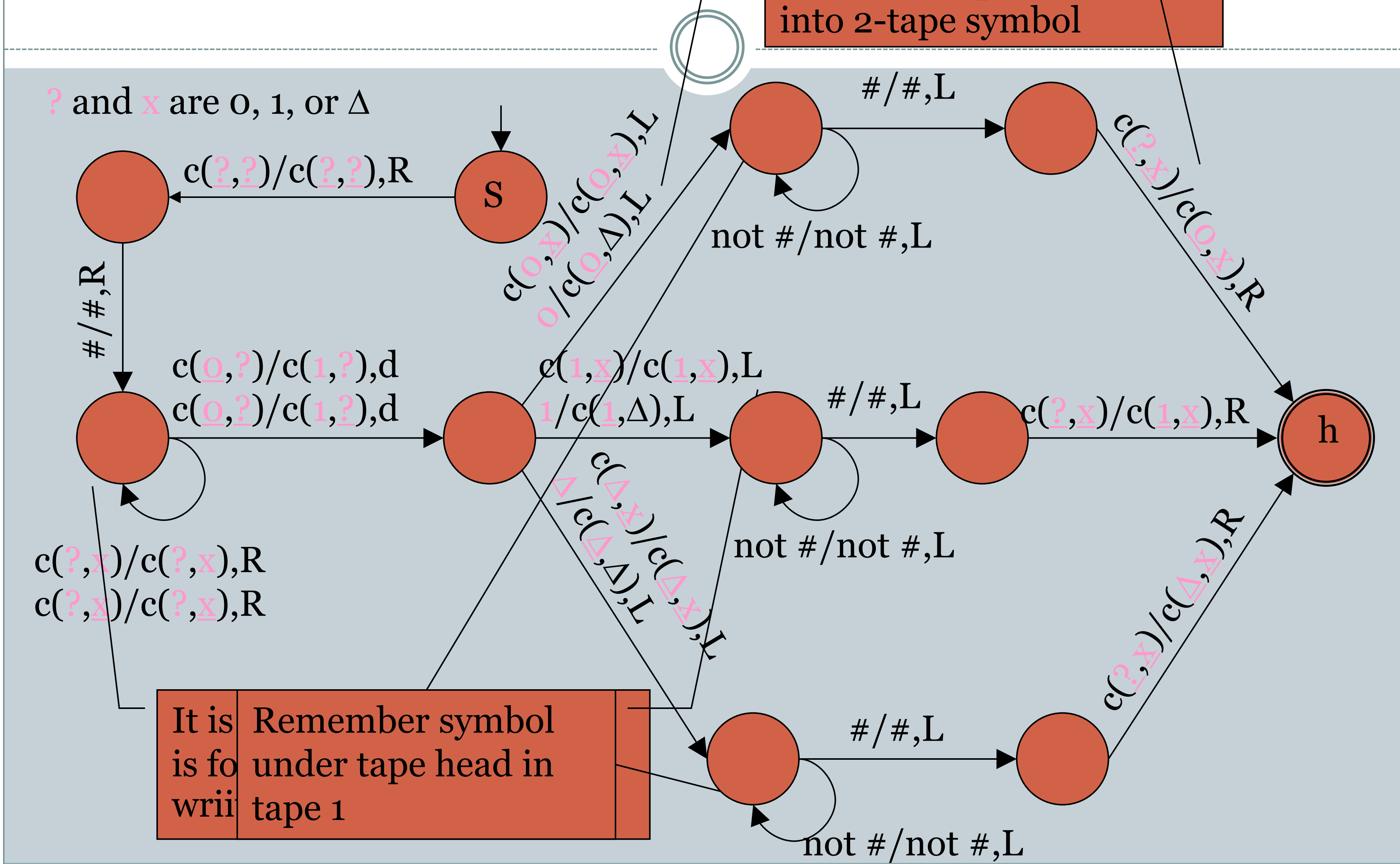
Tape format


c(b,Δ)
#
c(a,Δ)
c(b,Δ)
c(c,Δ)
c(d,Δ)
…
Encoded tape content
seperator
What's read on tape 1 and 2

Simulating transitions in 2-tape TM in 1-tape TM

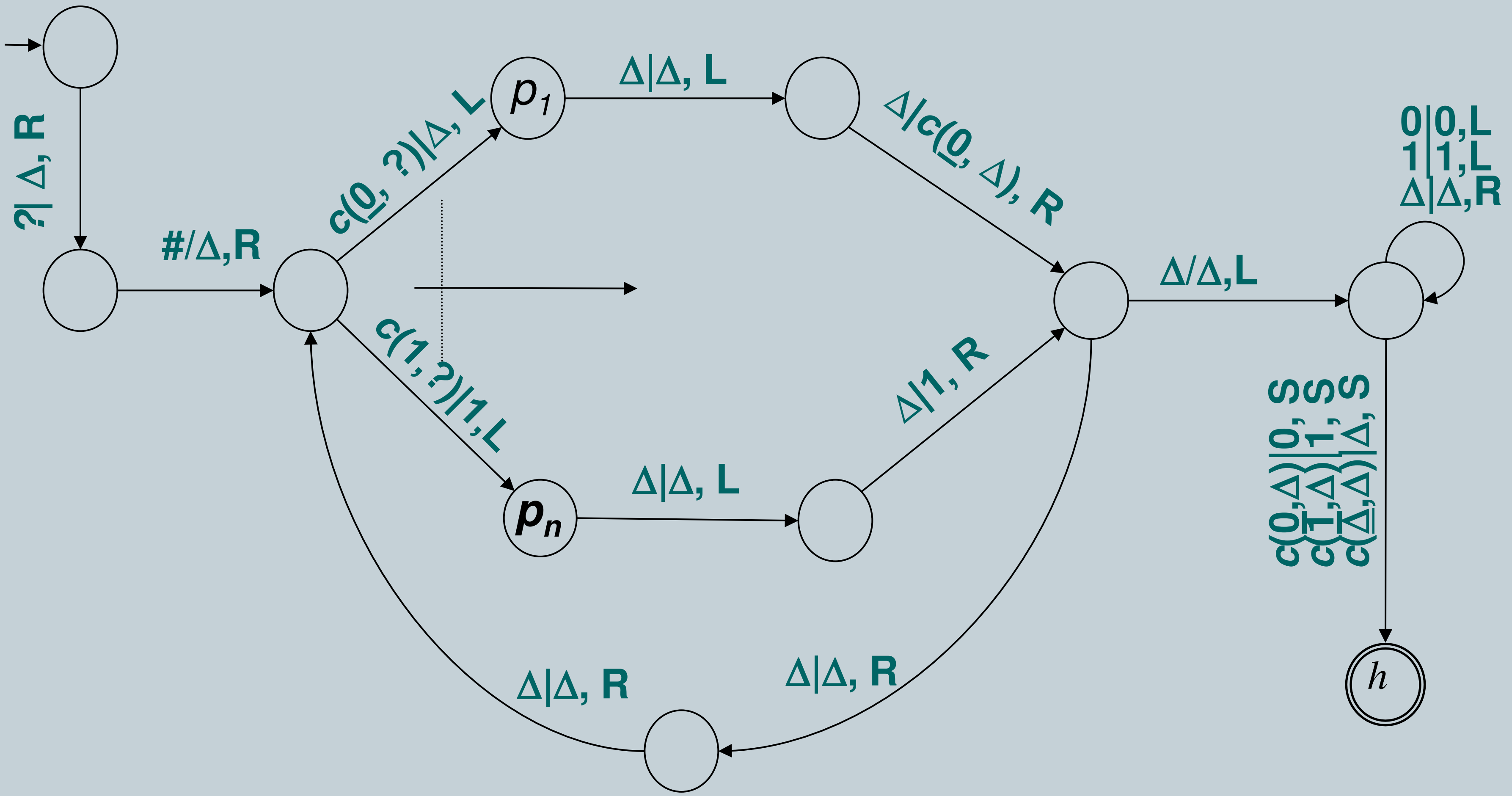


T_tape1(0,1,d)

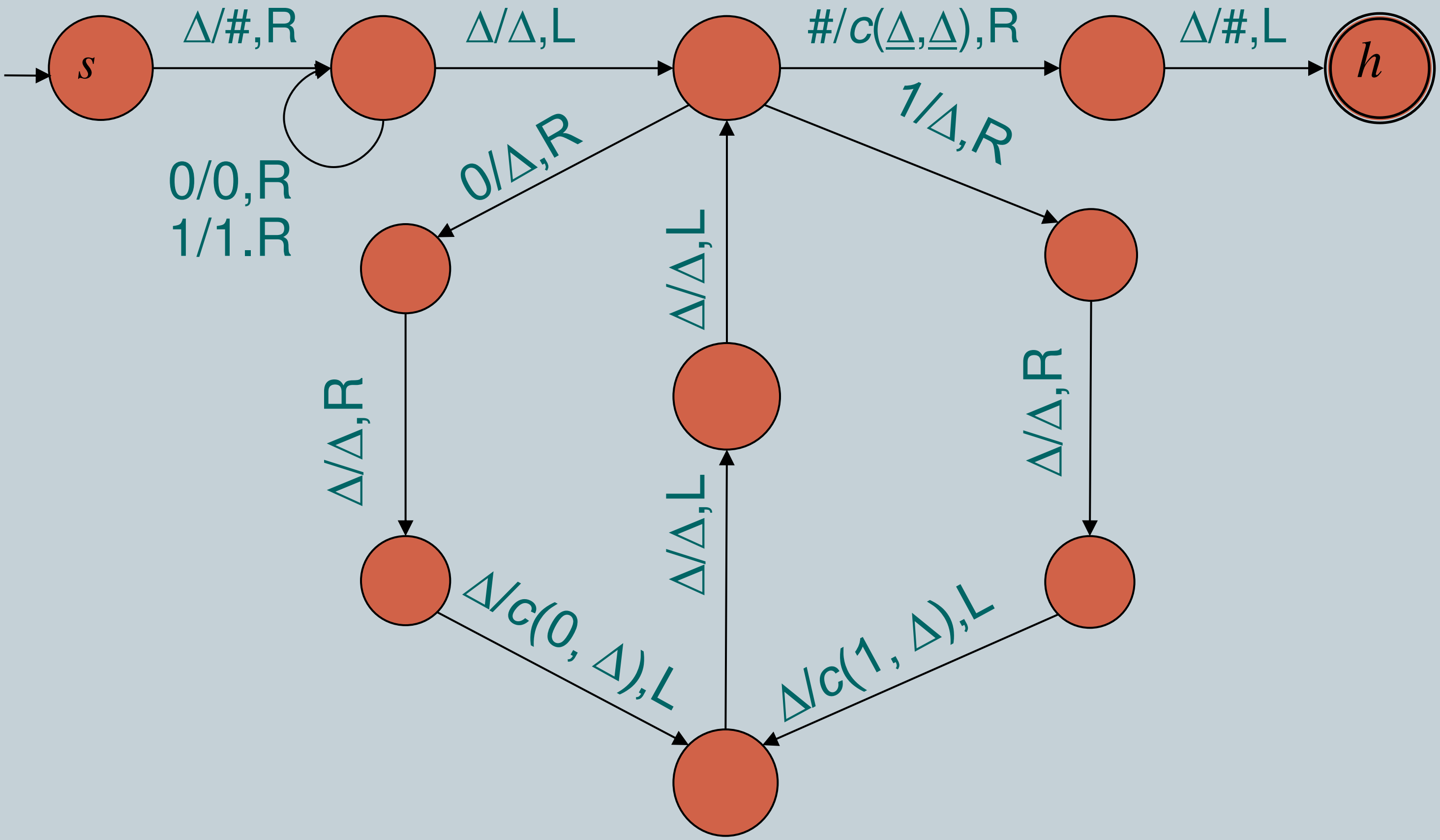


Update the first cell
Convert 1-tape symbol into 2-tape symbol
? and x are 0, 1, or Δ
c(?,?)/c(?,?),R
S
#/#,R
c(0,x)/c(0,x),L
0/c(0,Δ),L
#/#,L
not #/not #,L
c(?,x)/c(0,x),R
c(0,?)/c(1,?),d
c(0,?)/c(1,?),d
c(1,x)/c(1,x),L
1/c(1,Δ),L
#/#,L
c(?,x)/c(1,x),R
h
c(Δ,x)/c(Δ,x),L
Δ/c(Δ,Δ),L
not #/not #,L
c(?,x)/c(?,x),R
c(?,x)/c(?,x),R
c(?,x)/c(Δ,x),R
#/#,L
not #/not #,L
It is
is fo
wrii
Remember symbol under tape head in tape 1

T_{cleanup}



T_{encode}



Equivalence of 2-tape TM and single-tape TM

Proof:

Let $T = (Q, \Sigma, \Gamma, \delta, s)$ be a 2-tape TM.

We construct a 1-tape TM $M=(K, \Sigma, \Gamma', \delta', s')$ such that

- $\Gamma' = \Gamma \cup \{c(a,b) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{c(\underline{a},b) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{c(a,\underline{b}) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{c(\underline{a},\underline{b}) \mid a,b$ are in $\Gamma \cup \{\Delta\}\} \cup \{\#\}$

We need to prove that:

- if T halts on α with output β, then M halts on α with output β, and
- if T does not halt on α, then M does not halt on α

if T does not halt on α

- If T loops, then M loops.
- If T hangs in a state p, M hangs somewhere from p to the next state.